

February 1, 2008

Light-Front QCD(1+1) Coupled to Adjoint Scalar Matter

Stephen S. Pinsky

*Department of Physics
The Ohio State University
174 West 18th Avenue
Columbus, OH 43210*

Alex C. Kalloniatis

*Max-Planck-Institut für Kernphysik
Postfach 10 39 80
D-69029 Heidelberg*

Abstract

We consider adjoint scalar matter coupled to QCD(1+1) in light-cone quantization on a finite ‘interval’ with periodic boundary conditions. We work with the gauge group $SU(2)$ which is modified to $SU(2)/Z_2$ by the non-trivial topology. The model is interesting for various nonperturbative approaches because it is the sector of zero transverse momentum gluons of pure glue QCD(2+1), where the scalar field is the remnant of the transverse gluon component. We use the Hamiltonian formalism in the gauge $\partial_- A^+ = 0$. What survives is the dynamical zero mode of A^+ , which in other theories gives topological structure and degenerate vacua. With a point-splitting regularization designed to preserve symmetry under large gauge transformations, an extra A^+ dependent term appears in the current J^+ . This is reminiscent of an (unwanted) anomaly. In particular, the gauge invariant charge and the similarly regulated P^+ no longer commute with the Hamiltonian. We show that nonetheless one can construct physical states of definite momentum which are not *invariant* under large gauge transformations but do *transform* in a well-defined way. As well, in the physical subspace we recover vanishing *expectation values* of the commutators between the gauge invariant charge, momentum and Hamiltonian operators. It is argued that in this theory the vacuum is nonetheless trivial and the spectrum is consistent with the results of others who have treated the large N , $SU(N)$, version of this theory in the continuum limit.

1 Introduction

The unique features of ‘front form’ or light-cone quantized field theory provide a powerful tool for the study of QCD. Of primary importance in this approach is the existence of a vacuum state that is the ground state of the full theory. The existence of this state gives a firm basis for the investigation of many of the complexities that must exist in QCD. In this picture the rich structure of vacuum is transferred to the zero modes of the theory. Within this context the long range physical phenomena of spontaneous symmetry breaking [1] as well as the topological structure of the theory [2] can be associated with the zero mode(s) of the fields in a quantum field theory defined in a finite spatial volume and quantized at equal light-cone time [3].

These phenomena are realized in two quite different ways in several simpler theories. For example, spontaneous breaking of Z_2 symmetry in ϕ_{1+1}^4 occurs via a *constrained* zero mode [1]. There the zero mode satisfies a non-linear constraint equation that relates it to the dynamical modes in the problem [4]. At the critical coupling a bifurcation of the solution occurs. These solutions in turn lead to new operators in the Hamiltonian which break the Z_2 symmetry at and beyond the critical coupling. The work of Franke et al. [5] shows that such constrained zero modes are present in gauge theories, for example in (3+1) dimensions. Quite separately, a dynamical zero mode was shown in [2] to arise in pure $SU(2)/Z_2$ Yang-Mills in 1+1 dimensions. A complete fixing of the gauge leaves the theory with one degree of freedom, the zero or gauge mode of the vector potential A^+ . The theory has a discrete spectrum with zero momentum P^+ states corresponding to modes of the flux loop around the finite space. Only one state has an eigenvalue zero of energy, P^- , and is the true ground state of the theory. The non-zero eigenvalues are proportional to the length of the spatial box, consistent with the flux loop picture. This is a direct result of the topology of the space. As the theory considered there was a purely topological field theory the exact solution was identical to that in the ‘instant form’ approach on the analogous spatial topology [6].

In the present work we consider QCD_{1+1} coupled to scalar adjoint matter, also studied in the absence of zero modes by [7]. This theory can be obtained by dimensional reduction to (1+1) of pure glue theory in (2+1) dimensions. The scalar field is the remnant of the transverse gluon component. Our study of this theory is part of a long term program to attack QCD(3+1) through the zero modes sectors starting with studies of lower dimensional

theories which are themselves zero mode sectors of higher dimensional theories [2, 8]. A complete gauge fixing has recently been found for QED [8] which further supports this program. In all these cases, the goal was to disentangle the *dependent* from the *independent* degrees of freedom, in particular for the zero modes. As we showed in an earlier treatment [9], dimensionally reduced pure glue theory in (2+1) dimensions exhibits both types of zero modes. The dynamical zero mode comes from the (1+1) Yang-Mills sector while the constrained mode is in the scalar, namely remnant transverse gluon, field. In [10] a method for solving the, in this case, linear constraint was developed with the result that there is no vacuum degeneracy even though hints of how such degeneracy could take place appeared. We shall comment more on this in the conclusions. Here, we investigate the consequences of regulating currents using gauge-invariant point-splitting similar to that used by Manton [11] in the Schwinger Model and more recently by Lenz, Shifman and Thies [12] for QCD_{1+1} coupled to adjoint fermions. This regularization respects the symmetries of the theory under large gauge transformations and Weyl conjugations. The results are somewhat surprising: we find that an extra term is generated in the current J^+ whose diagonal color charge Q_3 itself is meant to generate global gauge transformations. This ‘anomalous’ term involves the zero mode of A^+ left after a complete gauge fixing of the theory. A similar term appears in the momentum P^+ operator. These contributions mean that symmetries such as Lorentz and charge invariance cannot be realized in a Hilbert space of states satisfying the large gauge symmetry. This problem is in fact generic to this type of treatment of any (1+1)-dimensional gauge theory on the light-cone. It is peculiar because this does not appear to occur in quantization on a space-like surface of the same theories. We propose that the resolution to the dilemma is to give up *invariance* of the ‘physical’ states under large gauge transformations. The naive normal ordered charge and momentum operators, which commute with the Hamiltonian, can be used to label the states. Because large gauge transformations can be realized in the quantum theory as unitary transformations of the Hilbert space, the physics, such as the spectrum of eigenvalues of the mass-squared operator, is invariant.

In section II we formulate the general structure of the theory in $SU(2)/Z_2$. In section III we quantize the model and introduce the zero mode structure of the theory. We discuss the symmetries of the theory. In Section IV we perform the point-splitting regularisation for Q_3 and P^+ . In Section V we discuss our results and contrast it with the treatments in [9, 10].

2 Dimensional Reduction of Pure Glue Theory

In the following we briefly reiterate the formulation we presented first in [9]. We take the pure Yang-Mills Lagrangian density

$$\mathcal{L} = -\frac{1}{2}\text{Tr}(\mathbf{F}^{\mu\nu}\mathbf{F}_{\mu\nu}) \text{ , with } \mathbf{F}^{\mu\nu} \equiv \partial^\mu \mathbf{A}^\nu - \partial^\nu \mathbf{A}^\mu + ig[\mathbf{A}^\mu, \mathbf{A}^\nu] \quad (2.1)$$

for which the energy-momentum tensor is $\Theta^{\mu\nu} = 2\text{Tr}(\mathbf{F}^{\mu\kappa}\mathbf{F}_\kappa{}^\nu) - g^{\mu\nu}\mathcal{L}$. In the front form, it is convenient to split the latter and their Lorentz indices $\mu(\nu)$ into the longitudinal values $\alpha(\beta) = +, -$ and into the transversal components. We use the convention $x^\pm = (x^1 \pm x^2)/\sqrt{2}$ and $A^+ = A^-$. The Lagrangian and the energy-density Θ^{+-} then disentangle nicely and in (2+1) dimensions we find, $\mathcal{L} = -\frac{1}{2}\text{Tr}(\mathbf{F}^{\alpha\beta}\mathbf{F}_{\alpha\beta} + 2\mathbf{F}^{\alpha j}\mathbf{F}_{\alpha j})$ and $\Theta^{+-} = \text{Tr}(\mathbf{F}^{-+}\mathbf{F}^{-+})$. The original formulation of Discretized Light-Cone Quantization (DLCQ) [3] was formulated in terms of only the normal modes. Here we pursue a complementary approach and analyze the theory in terms of only the zero modes. In particular we consider the theory with only transverse zero modes by requiring $\partial_i \mathbf{A}^\mu = 0$. Since all fields are thus independent of the transverse coordinate it is convenient to readjust units by scaling out the transverse length L_\perp . An adjustment of notation, $\mathbf{A}^\mu = (\mathbf{A}^+, \mathbf{A}^-, \mathbf{A}^1) \equiv (\mathbf{V}, \mathbf{A}, \Phi)$, helps to avoid too many indices. The model-theory then takes the form of a (1+1) gauge theory covariantly coupled to an adjoint scalar matter field [7],

$$\mathcal{L} = \text{Tr}\left(-\frac{1}{2}\mathbf{F}^{\alpha\beta}\mathbf{F}_{\alpha\beta} + \mathbf{D}^\alpha\Phi\mathbf{D}_\alpha\Phi\right) . \quad (2.2)$$

The equations of motions are correspondingly

$$\mathbf{D}_\beta\mathbf{F}^{\beta\alpha} = g\mathbf{J}^\alpha \text{ , with } \mathbf{J}^\alpha = -i[\Phi, \mathbf{D}^\alpha\Phi] \text{ , and } \mathbf{D}^\alpha\mathbf{D}_\alpha\Phi = 0 . \quad (2.3)$$

The currents \mathbf{J}^α are introduced for convenience and are only covariantly conserved. To fix the gauge, we follow the procedure given in [2] and find that A^+ only has a zero mode, i.e. $\partial_- \mathbf{A}^+ = 0$. After a rotation in color space, the matrix is diagonal in color space. In the instant form this gauge has been used by [6, 13] to name a few. In a context related to the front form it has also been used by [14].

In the present notation $\mathbf{F}^{-+} = \partial_+ \mathbf{V} - \mathbf{D}_- \mathbf{A}$. The first of our three equations of motion in the gauge sector is thus simply Gauss' law, $\mathbf{D}_- \mathbf{F}^{-+} = -\mathbf{D}_-^2 \mathbf{A} = g\mathbf{J}^+$, realized here as a second class constraint in the nomenclature of Dirac. In the absence of gauge-fixing these are first class constraints and are a consequence of the gauge-symmetry. With the

gauge-fixing, these can be realized as quantum operator constraints. The off-diagonal part of this equation can be solved strongly yielding

$$\mathbf{A} = -g \frac{1}{\mathbf{D}_-^2} \mathbf{J}^+ . \quad (2.4)$$

The diagonal projection of the zero mode part of Gauss's law remains first class. This must be satisfied as a condition on the states, *i.e.* $\langle (\mathbf{J}^+)_{\text{diag}} \rangle_{\circ} |\text{phys}\rangle = 0$. Analogous constraints can be found in other contexts, eg [15]. Here $\langle f \rangle_{\circ}$ is the zero mode projection $\int_{-L}^L dx f(x)/2L$ of some quantity $f(x)$, as in our earlier work, eg [9]. Since we pursue a Hamiltonian approach we do not give the detailed expressions for the genuinely dynamical equations. Sufficeth to say, they exist for the gauge mode \mathbf{V} and for the scalar field. The zero mode projection of the color-diagonal part of the latter equation will be shown to generate a constraint equation. In the Dirac procedure it occurs as a second class constraint.

Finally, the formal expression for the Hamiltonian is

$$P^- = \int_{-L}^{+L} dx^- \text{Tr} (\partial_+ \mathbf{V} - \mathbf{D}_- \mathbf{A})^2 = \int_{-L}^{+L} dx^- \text{Tr} (\partial_+ \mathbf{V} \partial_+ \mathbf{V} - g^2 \mathbf{J}^+ \frac{1}{\mathbf{D}_-^2} \mathbf{J}^+) . \quad (2.5)$$

It describes the interaction of two matter currents of adjoint scalars via an instantaneous gluon-like interaction [7]. The instantaneous gluon is modified by the zero mode of \mathbf{A}^+ . This zero mode structure produces an effective mass [16] and in no case is $1/\mathbf{D}_-^2$ singular.

3 Quantization and Matter Currents

In the following we will use a color helicity basis for all field matrices of the form

$$\Phi = \tau^3 \varphi_3 + \tau^+ \varphi_+ + \tau^- \varphi_- \quad (3.6)$$

where $\tau^a = \sigma^a/2$ and $\tau^{\pm} = (\tau^1 \pm i\tau^2)/\sqrt{2}$. The zero mode matrix \mathbf{V} is diagonal, thus $\mathbf{V} = v \tau_3$, where $v \equiv v(x^+)$ is a quantum mechanical operator as discussed in [2, 11]. The conjugate momentum is $p \equiv \delta L/\delta v = 2L\partial_+ v$. It satisfies the commutation relation $[v, p] = [v, 2L\partial_+ v] = i$. Whenever we see the operator v in the subsequent analysis it is understood that we work in a representation which diagonalizes the operator v . Thus $p = -id/dv$, see [11].

The diagonal components of the hermitean matrix Φ is denoted by φ_3 . Any real-valued boson field subject to periodic boundary conditions can be represented by

$$\varphi_3(x^+, x^-) = \frac{a_0(x^+)}{\sqrt{4\pi}} + \frac{1}{\sqrt{4\pi}} \sum_{n=1}^{\infty} w_n \left(a_n(x^+) e^{-ik_n x^-} + a_n^\dagger(x^+) e^{+ik_n x^-} \right) . \quad (3.7)$$

where $k_n = 2\pi n/(2L)$ denote the discretized momenta. With $[a_n, a_m^\dagger] = \delta_{n,m}$ ($n, m = 1, \dots, \infty$) and coefficients $w_n = 1/\sqrt{n}$ one gets the correct commutation relations for field operators with *no* zero modes, *i.e.*

$$[\varphi_3(x), \pi^3(y)]_{x^+=y^+} = \frac{i}{2}(\delta(x^- - y^-) - \frac{1}{2L}) , \quad (3.8)$$

The momentum field conjugate to φ_3 is denoted by $\pi^3 \equiv \partial_- \varphi_3$. The ‘zero mode’ $a_0 = a_0^\dagger$, however, obeys a constraint equation obtained by the projection $\text{Tr} \langle \tau^3 \mathbf{D}^\alpha \mathbf{D}_\alpha \Phi \rangle_\circ = 0$. As we observed in our previous work [9], this constraint is *linear* in a_0 , which appears through the currents J_\pm^\pm . Thus this is quite different in structure from the constraint equation of the ϕ_{1+1}^4 theory [1]. We return to this in the final discussion.

The off-diagonal components of Φ are complex valued fields, $\varphi_+(x^+, x^-) = \varphi_-^\dagger(x^+, x^-)$. Any such boson field subject to periodic boundary conditions can be written as

$$\varphi_-(x^+, x^-) = \frac{1}{\sqrt{4\pi}} \left(\sum_{n=0}^{\infty} b_n u_n e^{-ik_n x^-} + \sum_{n=1}^{\infty} d_n^\dagger v_n e^{+ik_n x^-} \right) . \quad (3.9)$$

The analogy with the (complex) Dirac spin components is convenient for discussing some of the symmetries of the theory. In the following we take a somewhat different approach to our treatment in [9]. The canonical momentum fields conjugate to φ_- and φ_+ are $\pi^- = (\partial_- + igv)\varphi_+$ and $\pi^+ = (\partial_- - igv)\varphi_-$ and these satisfy equal x^+ commutation relations with the fields,

$$[\varphi_-(x^-), \pi^-(y^-)] = [\varphi_+(x^-), \pi^+(y^-)] = \frac{i}{2}\delta(x^- - y^-) . \quad (3.10)$$

These relations can be satisfied with the choice of coefficients

$$u_n \equiv \frac{1}{\sqrt{|n+z|}} , \quad \text{and} \quad v_n \equiv \frac{1}{\sqrt{|n-z|}} , \quad \text{with} \quad z \equiv \frac{gvL}{\pi} \quad (3.11)$$

and with commutation relations for the b, d operators

$$[b_n, b_m^\dagger] = \text{sgn}(n+z)\delta_{n,m}, \quad [d_n, d_m^\dagger] = \text{sgn}(n-z)\delta_{n,m}, \quad [b_n, d_m] = [b_n, d_m^\dagger] = 0 . \quad (3.12)$$

Note that the assignment of creation and annihilation operator depends on z . With these results it is useful to express the conjugate momentum expansion as

$$\pi^+(x^+, x^-) = \frac{-i}{4L} \sqrt{4\pi} \left(\sum_{n=0}^{\infty} b_n \frac{\text{sgn}(n+z)}{u_n} e^{-ik_n x^-} - \sum_{n=1}^{\infty} d_n^\dagger \frac{\text{sgn}(n-z)}{v_n} e^{+ik_n x^-} \right) . \quad (3.13)$$

with $\pi^- = (\pi^+)^\dagger$. The result that $(n+z)u_n = (n+z)/\sqrt{n+z} = \text{sgn}(n+z)/u_n$ is useful for obtaining this result. One should also observed that in the above we have made a choice at the edge of the “Dirac-sea” to assign to the dynamical zero mode of φ_- a b_0 rather than a d_0^\dagger operator. One could write it as a superposition but a trivial Bogoliubov transformation allows one to transform the vacuum and states between different choices.

Because of the torus geometry of our space and the non-Abelian structure of the gauge group, there remain large gauge transformations which are still symmetries of the theory [17]. We have explained how to completely fix the gauge with respect to small gauge transformations. The large gauge transformations are generated by local $SU(2)/Z_2$ elements $V(x) = \exp(-i\frac{\pi}{L}x \tau_3)$, which is anti-periodic. On the diagonal component v it generates shifts that are best expressed in terms of the dimensionless z : $z \rightarrow z' = z + 1$. A shift by any integer is generated by repeated application of this transformation. On the scalar adjoint fields, the effect of the transformation is

$$\varphi_3 \rightarrow \varphi'_3 = \varphi_3 \quad (3.14)$$

$$\varphi_\pm \rightarrow \varphi'_\pm = \varphi_\pm \exp(\mp i\frac{\pi}{L}x). \quad (3.15)$$

This leads to the following effect on the modes b and d

$$d_1^\dagger \rightarrow b_0 \quad (3.16)$$

$$d_n^\dagger \rightarrow d_{n-1}^\dagger \quad n \geq 2 \quad (3.17)$$

$$b_n \rightarrow b_{n+1} \quad n \geq 0. \quad (3.18)$$

As a consequence we find a spectral flow very similar to the problem with fermions [11, 12]: some of the hole states are elevated to occupied particle states. However, the physical content of any given domain $M \leq z \leq M+1$ is the same for all M . We shall label the domains by the integer M .

The theory has an additional Weyl conjugation symmetry, $z \rightarrow z' = -z$. On the b and d modes the transformation is similar to charge conjugation

$$b_n \leftrightarrow d_n \quad n \geq 1, b_0 \leftrightarrow b_0^\dagger. \quad (3.19)$$

We see that the Weyl conjugation preserves the commutation relations in an interesting way. The factor $\text{sgn}(n+z)$ in the commutation relations changes sign for $n=0$ when $z \rightarrow -z$ and the interchange of $b_0 \leftrightarrow b_0^\dagger$ compensates for it. This symmetry also introduces a degeneracy in each domain, which we label by the integer M . The lower half of the

domains, $M \leq z \leq M + \frac{1}{2}$ are related to the upper half $M + \frac{1}{2} \leq z \leq M + 1$ of the domains. To see this consider the $M = 0$ domain, the fundamental modular domain (FMD). The region $0 \leq z \leq \frac{1}{2}$ is equivalent to the region $-\frac{1}{2} \leq z \leq 0$ by Weyl conjugation and this region is equivalent to the region $\frac{1}{2} \leq z \leq 1$ by a large gauge transformation. This in effect forces the domain to be symmetric about $z = \frac{1}{2}$. In [12] was shown that it is this symmetry about $\frac{1}{2}$ that gives QCD_{1+1} coupled to adjoint fermions a degenerate vacuum. In that model one vacuum wave function is centered just above $z = 0$ and the other is centered just below $z = 1$.

To close this section we repeat the argument in [9] showing that the gauge mode z can be written in terms of an explicitly color singlet object, namely the Wilson loop constructed via a contour C along the x direction from $-L$ to L :

$$W = \text{TrP} \exp(ig \int_C dx_\mu \mathbf{A}^\mu) = \text{TrP} \exp(ig \int_{-L}^{+L} dx \mathbf{A}^+) = \text{Tr} \exp(2i z \pi \tau^3). \quad (3.20)$$

Thus we can relate z to W modulo the integers, $z = \frac{1}{2\pi} \arccos(\frac{W}{2})$. The integer shifts are nothing but the Gribov copies discussed earlier. Observe that the dynamical quantity W attains its minimum value at $z = \frac{1}{2}$ making the point $z = \frac{1}{2}$ the symmetry point in the FMD.

4 Point-Splitting

We begin by rewriting Gauss' law in components, *i.e.*

$$-\partial_-^2 A_3 = gJ_3^+, \quad -(\partial_- + igv)^2 A_+ = gJ_+^+, \quad (4.21)$$

and the hermitian conjugate of the latter with $(J_+^+)^{\dagger} \equiv J_-^+$. One would like to invert these to express A_3 and A_{\pm} in terms of the currents J^+ which, according to Eq.(2.3), are defined as

$$J_3^+ = \frac{1}{i}(\varphi_+ \pi_- - \varphi_- \pi_+) \text{ and } J_+^+ = \frac{1}{i}(\varphi_3 \pi_+ - \varphi_+ \pi_3). \quad (4.22)$$

The first of the Gauss equations (4.21) can be solved only if the zero mode $\langle J_3^+ \rangle_0$ on the r.h.s vanishes also. This cannot be satisfied as an operator, but rather as a condition on the physical states, *i.e.* $\langle J_3^+ \rangle_0 | \text{phys} \rangle \equiv 0$. The calculation of the currents requires some care since it involves the difference of the product of operators at the same point. We regulate the divergent sums by a gauge-invariant point-splitting.

$$J_3^+ = \lim_{\varepsilon \rightarrow 0} \frac{1}{i} \left[\varphi_+(x^- - \varepsilon) \pi_-(x^-) e^{-igv\varepsilon} - \varphi_-(x^- - \varepsilon) \pi_+(x^-) e^{+igv\varepsilon} \right] \quad (4.23)$$

This gives the follow form for the charge operator before performing the sums or taking the limit

$$Q_3 = -\text{sgn}(z)b_0^\dagger b_0 \cos(\varepsilon\pi z/L) - \frac{1}{2}e^{i\varepsilon\pi z/L} + \sum_{n=1}^{\infty} \left(-\text{sgn}(n+z)b_n^\dagger b_n \cos(\varepsilon\pi(n+z)/L) - \frac{1}{2}e^{i\varepsilon\pi(n+z)/L} + \text{sgn}(n-z)d_n^\dagger d_n \cos(\varepsilon\pi(n-z)/L) + \frac{1}{2}e^{i\varepsilon\pi(n-z)/L} \right). \quad (4.24)$$

Performing the sums and taking the limit produces the following expression for Q_3 :

$$Q_3 = -\sum_{n=0}^{\infty} \text{sgn}(n+z)b_n^\dagger b_n + \sum_{n=1}^{\infty} \text{sgn}(n-z)d_n^\dagger d_n + \left(z - \frac{1}{2}\right). \quad (4.25)$$

It is straightforward to show that the operator Q_3 is symmetric under large gauge transformations and antisymmetric under Weyl conjugation. We see the appearance of the anomalous term z in Q_3 . It is easy to see that because of the kinetic term for the gauge mode in the Hamiltonian Eq.(2.5), the gauge invariant regularized charge does not commute with the Hamiltonian. It is important to point out that this does not occur in a similar treatment of the charge operator in conventional quantization, say, of the Schwinger model [11].

We can also calculate P^+ ,

$$P^+ = \int_{-L}^L dx^- (\pi_3 \pi_3 + \pi_+ \pi_- + \pi_- \pi_+). \quad (4.26)$$

This operator involves operator products at the same point and requires the same careful treatment used to calculate Q_3 . We find,

$$P^+ = \frac{\pi}{L} \left(\sum_{n=1}^{\infty} (n a_n^\dagger a_n + |n-z|d_n^\dagger d_n + |n+z|b_n^\dagger b_n) + |z|b_0^\dagger b_0 - \frac{1}{2}(z - \frac{1}{2})^2 - \frac{3L^2}{2\pi^2\varepsilon} \right). \quad (4.27)$$

Direct calculation shows that P^+ is symmetric under large gauge transformations and Weyl conjugation. The last term is a divergent constant and can be trivially renormalized. The term proportional to $(z - \frac{1}{2})^2$ however destroys the commutativity of the momentum with the Hamiltonian.

One should also regulate noncommuting operator products in P^- using point-splitting. This is too lengthy to treat here. It suffices to say that the gauge factor introduced in the splitting could not generate the terms required to maintain the commutation relations between all the operators: only an extra term with the conjugate momentum of the gauge mode can help recover the vanishing commutators, and this cannot arise from point-splitting.

We can however define an operator \tilde{Q}_3 via

$$Q_3 \equiv \tilde{Q}_3 + (z - \frac{1}{2}) \quad (4.28)$$

for which $[\tilde{Q}_3, P^-] = 0$. Moreover, we can similarly relate the regulated momentum operator to a ‘naive’ momentum operator \tilde{P}^+ via

$$P^+ \equiv \tilde{P}^+ + z\tilde{Q}_3 - \frac{1}{2}(z - \frac{1}{2})^2 \quad (4.29)$$

after subtracting the divergent constant. One can then show that $[\tilde{P}^+, P^-] = 0$.

We have thus succeeded in constructing representations of the charge and momentum operators in terms of a Fock space implementing their symmetries with the Hamiltonian. Evidently, the operators are not invariant under the large gauge symmetries so that consequently the Hilbert space is not invariant. The states can be labelled by

$$|N_b; N_d; z\rangle = \Psi(z)|N_b; N_d\rangle \quad (4.30)$$

reflecting the Fock-mode content as well as that of the gauge mode in its ground state. As mentioned in the introduction, only the ground state wavefunction of the gauge mode contributes to the tensor product in the continuum limit [2]. Among all of these states we will restrict the physical states to be those where $N_b = N_d$ in the FMD, $0 < z < 1$ such that they are annihilated by the non-invariant charge operator \tilde{Q}_3 . Under large gauge transformations these states transform. For example, under $z \rightarrow z + 1$, the charge \tilde{Q}_3 transforms $\tilde{Q}_3 \rightarrow \tilde{Q}_3 - 1$. Thus if we represent the transformation by a unitary operator T and consider a physical state defined in the FMD by $|N; N; z\rangle$, then $\tilde{Q}_3 T^\dagger |N; N; z\rangle = T |N; N; z\rangle$. Thus the transformed state is a state with an extra d -mode. As the transformation can be represented by a unitary operator on the Hilbert space constructed in the FMD the spectrum of the mass-squared operator will be invariant.

In fact one can show that the expectation value of the fully gauge invariant Q_3 vanishes between any Fock state with equal numbers of b and d modes tensored with the ground state of the gauge mode. Since the theory is symmetric about $z = \frac{1}{2}$ the wavefunctional will be either symmetric or antisymmetric. We now take the expectation value

$$\int_0^1 dz |\Psi(z)|^2 \langle N; N | Q_3 | N'; N' \rangle = \delta_{N, N'} \int_0^1 dz |\Psi(z)|^2 (z - \frac{1}{2}) \quad (4.31)$$

which vanishes due to the product of the antisymmetric $(z - \frac{1}{2})$ with the symmetric $|\Psi(z)|^2$ under the integration. Similarly, one can show that the commutators $[P^-, Q_3]$ and $[P^-, P^+]$ vanish in the sense of expectation values.

5 Discussion and Conclusions

We considered the transverse zero mode sector of QCD_{2+1} . The theory manifests itself as QCD_{1+1} coupled to adjoint scalar matter which has symmetries with respect to large gauge transformations and Weyl conjugation. There were two zero modes upon complete gauge fixing. One, the longitudinal zero mode of φ_3 , is constrained. The other, the zero mode of A^+ , is a dynamical field. In the approach we took here, we generalized our previous method of quantizing the theory. In [9, 10] the classical Weyl and large gauge symmetries were not implemented in the quantum theory, but rather extracted as classical phases in the field expansions employed there. This actually was rather convenient in that it permitted a naive cutoff regularization which did not violate this symmetry in the resulting quantum theory. Here we have realized the large gauge transformations in the quantum sense as well. The theory was regularized using gauge-invariant point-splitting. In this manner it became impossible to construct a Fock representation of the charge and momentum operators respecting their commutativity with the Hamiltonian and, simultaneously, maintaining the symmetry under large gauge transformations. We found we could label states as eigenstates of the parts of Q_3 and P^+ which transform under the large gauge transformations. Correspondingly the states themselves transform in this approach. In this way physical quantities, such as the spectrum, remain invariant.

We now briefly discuss the significance of this work in relation to the two previous papers [9] and [10]. In [9], the potential governing the behaviour of the gauge mode was computed explicitly. However, there remained a logarithmic divergence. In [10], it was shown that this divergence could be removed by mass renormalisation. The resulting potential is consistent with the symmetry about $z = \frac{1}{2}$, argued in the present work by looking at the Wilson loop. As mentioned, the quantization in [9, 10] was performed keeping the large gauge symmetry as a classical phase in the field expansions. In this sense the spectral flow was not implemented in the quantum Hilbert space. It is not evident in the formulation of [9, 10] how, in a theory with a chiral anomaly, that approach can give the correct result, given the picture of [11] relating the anomaly to spectral flow in QED on a circle. Our findings here fill in that gap while also recovering the result of [9, 10]. Work applying these methods to both the Schwinger model and QCD(1+1) with adjoint fermions is in progress [18].

As for the spectrum of the theory, in the absence of the analogue of θ -vacua it would appear that the impact of the gauge mode becomes minimal in the continuum limit. The

gauge mode wavefunctions are essentially just sines or cosines, while in Fock-space matrix elements the gauge mode only appears in denominators such as $1/(n+z)$ or $1/(n-z)$. Consequently, in the limit of large harmonic resolution K , essentially large integer momentum, the impact of the gauge mode becomes small as has been observed in [19]. In the absence of any richer structures in this theory we are thus led back to the analogue of the original formulation by Klebanov et al. [7]. The possibility remains open that extension of this theory to include more scalar fields and fermions, as would arise by a dimensional reduction of QCD(3+1), would introduce enough richness so that the zero mode sector plays a more significant role in the final spectrum of the theory.

6 Acknowledgments

The authors acknowledge G. McCartor, Y. Frishman, B. van de Sande, K. Harada and D.G. Robertson for advice. (SSP) acknowledges the support of the U.S. Department of Energy. (ACK) was supported by the DFG under contract DFG-Gz: Pa 450/1-2. Travel support was provided in part by a NATO Collaborative Grant.

References

- [1] D.G. Robertson, *Phys. Rev.* **D47** (1993) 2549; T. Heinzl, S. Krusche and E. Werner, *Phys. Lett.* **B256** (1991) 55; C.M. Bender, S.S. Pinsky, B. van de Sande, *Phys. Rev.* **D48** (1993) 816; S.S. Pinsky and B. van de Sande, *Phys. Rev.* **D49** (1994) 2001; J. Hiller, S.S. Pinsky and B. van de Sande, *Phys. Rev.* **D51** (1995) 726;
- [2] A.C. Kalloniatis, H.C. Pauli, and S.S. Pinsky, *Phys. Rev.* **D50** (1994) 6633.
- [3] H.C. Pauli and S.J. Brodsky, *Phys. Rev.* **D32** (1985) 1993; *Phys. Rev.* **D32** (1985) 2001.
- [4] T. Maskawa and K. Yamawaki, *Prog. Theor. Phys.* **56** (1976) 270.
- [5] V.A. Franke, Yu.A. Novozhilov, and E.V. Prokhvatilov, *Lett. Math. Phys.* **5** (1981) 239; 437. (Academic Press, 1982, p.389-400).
- [6] J.E. Hetrick, *Nucl. Phys.* **B30** (1993) 228.
- [7] K. Demeterfi, I.R. Klebanov, and G. Bhanot, *Nucl. Phys.* **B418** (1994), 15; G. Bhanot, K. Demeterfi, and I.R. Klebanov, *Phys. Rev.* **D48** (1993) 4980.

- [8] A.C. Kalloniatis and H.C. Pauli, *Z.Phys.***C60** (1993) 255; *Z.Phys.***C63** (1994) 161;
A.C. Kalloniatis and D.G. Robertson, *Phys.Rev.* **D50** (1994) 5262.
- [9] H.-C. Pauli, A.C. Kalloniatis, and S.S. Pinsky, Heidelberg preprint MPI H-V3-1995
(Revised), hep-th/9509020. To appear in *Phys.Rev.* D.
- [10] A.C. Kalloniatis, Heidelberg preprint MPI-H-V29-1995, hep-th/9509027.
- [11] N.S. Manton, *Ann.Phys.(N.Y.)***159** (1985) 220.
- [12] F. Lenz, M. Shifman and M. Thies, *Phys.Rev.* **D51** (1995) 7060.
- [13] E. Langmann and G. W. Semenoff, *Phys. Lett.***B296** (1992), 117; E. Langmann,
M. Salmhofer, and A. Kovner, *Mod.Phys.Lett.* **A9** (1994) 2913.
- [14] F. Lenz, M. Thies, S. Levit, and K. Yazaki, *Ann.Phys.(N.Y.)***208** (1991) 1.
- [15] J.A. Minahan and A.P. Polychronakos, *Phys.Lett.***B326** (1994) 288.
- [16] S. Pinsky, p.223, in “Theory of Hadrons and Light-Front QCD”, Ed. St. Glazek, World
Scientific (Singapore) 1995.
- [17] V.N. Gribov, *Nucl. Phys.***B** 139 (1978) 1.
- [18] S.S. Pinsky, K. Harada, D.G. Robertson, manuscript in preparation.
- [19] H.-C. Pauli, R. Bayer, Heidelberg preprint MPI H-V32-1995.